Reversible Wavelets for Embedded Image Compression

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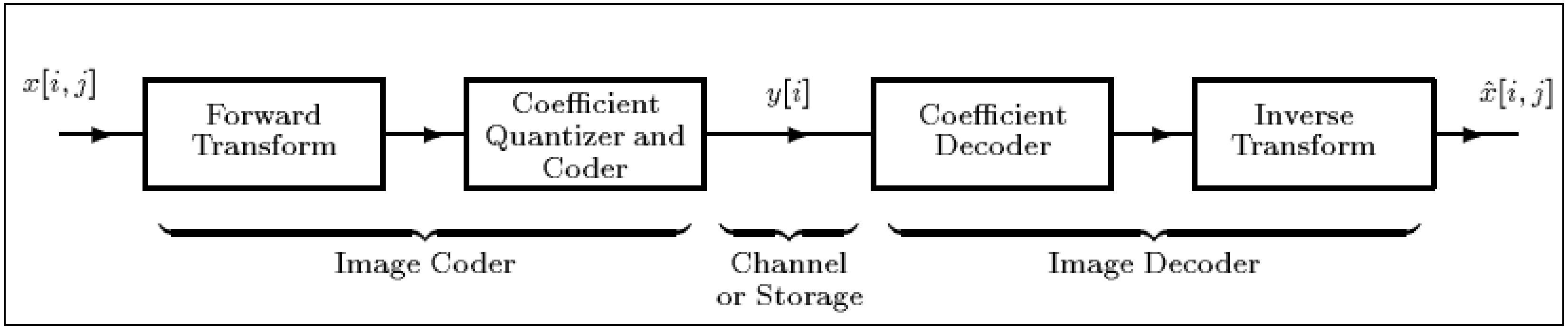
Introduction to Image Compression

Goal:

- Find an alternative representation to reduce storage space

Classifications:

- Lossless
- Lossy
- Spatial domain - Original image is used directly
- Transform based - Obtain coefficients that are easier to code



- Code symbols with higher probability of occurrence with fewer bits

Performance Measures

- Compression Ratio (CR) [> 1]:

$$\mathrm{CR} = \frac{\text{original image size in bits}}{\text{compressed image size in bits}}$$

- Bit Rate (BR) [bits per pixel]:

$$\mathrm{BR} = (\text{bits/pixel for original image})/\mathrm{CR}$$

Lossy compression distortion measures: [**M** x **N** image with **P** bits per pixel]

- Mean Squared Error (MSE):
  (Smaller the better)

$$\mathrm{MSE} = \frac{1}{MN}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}(x_{i,j} - \hat{x}_{i,j})^2$$

- Peak Signal to Noise Ratio (PSNR):
  (Greater the better)

$$\mathrm{PSNR} = 10\log_{10}\frac{(2^P - 1)^2}{\mathrm{MSE}}$$

MSE and PSNR do not always correlate with quality as perceived by the human eye!

Embedded Coding

Embedded Coding:

- Information in the coded bit stream is in the order of importance
- Lower bit rates found in the beginning of the bit stream
- Decoder can stop decoding at any point and the same image is obtained as would have been corresponding to the truncated bit stream

Application:

- Image browsing over low bit rate channels

Example:

- Embedded Zerotree Wavelet (EZW) Coding
  - Discrete wavelet transform
  - Zerotree coding of wavelet coefficients
  - Successive-approximation quantization (SAQ)
  - Adaptive arithmetic coding

J. M. Shapiro. An embedded wavelet hierarchical image coder. In *Proc. of IEEE International Conference on Acoustics, Speech, and Signal Processing*, number 4, pages 657–660, San Francisco, CA, March 1992.

Wavelets for Image Compression

Advantages:

- Many coefficients for a typical image are small (easy coding)
- Unlike DFT and DCT, basis functions can have both long and short support
- Long support - effective for representing slow variations
- Short support - effective for representing edges
- Wavelet decomposition results in frequency bands that are equally spaced on a logarithmic scale. Human visual perception behaves logarithmically too! (*Vision*, David Marr)

Ideal Characteristics:

- Orthogonal basis
- Symmetric basis

  - to avoid phase distortions that could result in distorted edges

  - to use symmetric expansion (better than periodic – *More later!)*
- Finite support
- M-band (M>2) wavelets offer superior compression performance

  - Coefficient decay rate = M^-kP [k=scale P=vanishing moments]

  - Orthogonal transforms with symmetric finitely-supported basis

Examples of Wavelet Transforms

Name	Description
Haar	Haar Transform $\leftrightarrow$ 2-band, orthogonal, symmetric, interpolating, 2/2, (1,1)
TS	Two-Six Transform $\leftrightarrow$ 2-band, biorthogonal, symmetric, 2/6, (3,1)
CDF22	Cohen-Daubechies-Feauveau (2,2) Transform $\leftrightarrow$ 2-band, biorthogonal, symmetric, interpolating, 5/3, (2,2)
CDF24	Cohen-Daubechies-Feauveau (2,4) Transform $\leftrightarrow$ 2-band, biorthogonal, symmetric, interpolating, 9/3, (2,4)
CDF97	Cohen-Daubechies-Feauveau 9/7 Transform $\leftrightarrow$ 2-band, biorthogonal, symmetric, 9/7, (4,4)
V610	Villasenor 6/10 Transform $\leftrightarrow$ 2-band, biorthogonal, symmetric, 6/10
MIT97	MIT 9/7 Transform $\leftrightarrow$ 2-band, biorthogonal, symmetric, interpolating, 9/7, (4,2)
BCW3	Biorthogonal Coifman Transform (Order 3) $\leftrightarrow$ 2-band, biorthogonal, symmetric, interpolating, 13/7, (4,4)
V4	Vaidyanathan 4-Band Transform $\leftrightarrow$ 4-band, orthogonal, symmetric
A4	Alkin 4-Band Transform $\leftrightarrow$ 4-band, orthogonal, symmetric

2-band analysis filters

$$\text{Haar}\quad\begin{cases} H_0(z) = \frac{1}{\sqrt{2}}(1+z^{-1}) \\ H_1(z) = \frac{-1}{\sqrt{2}}(1-z^{-1}) \end{cases}$$

$$\text{TS}\quad\begin{cases} H_0(z) = \frac{1}{\sqrt{2}}(1+z^{-1}) \\ H_1(z) = -\frac{1}{8\sqrt{2}}(1-z^{-5}) - \frac{1}{8\sqrt{2}}(z^{-1}-z^{-4}) + \frac{8}{8\sqrt{2}}(z^{-2}-z^{-3}) \end{cases}$$

$$\text{CDF22}\quad\begin{cases} H_0(z) = -\frac{1}{4\sqrt{2}}(1+z^{-4}) + \frac{2}{4\sqrt{2}}(z^{-1}+z^{-3}) + \frac{6}{4\sqrt{2}}z^{-2} \\ H_1(z) = \frac{1}{2\sqrt{2}}(1+z^{-2}) - \frac{2}{2\sqrt{2}}z^{-1} \end{cases}$$

$$\text{CDF24}\quad\begin{cases} H_0(z) = \frac{3\sqrt{2}}{128}(1+z^{-8}) - \frac{6\sqrt{2}}{128}(z^{-1}+z^{-7}) - \frac{16\sqrt{2}}{128}(z^{-2}+z^{-6}) + \frac{38\sqrt{2}}{128}(z^{-3}+z^{-5}) \\ \qquad + \frac{90\sqrt{2}}{128}z^{-4} \\ H_1(z) = \frac{\sqrt{2}}{4}(1+z^{-2}) - \frac{2\sqrt{2}}{4}z^{-1} \end{cases}$$

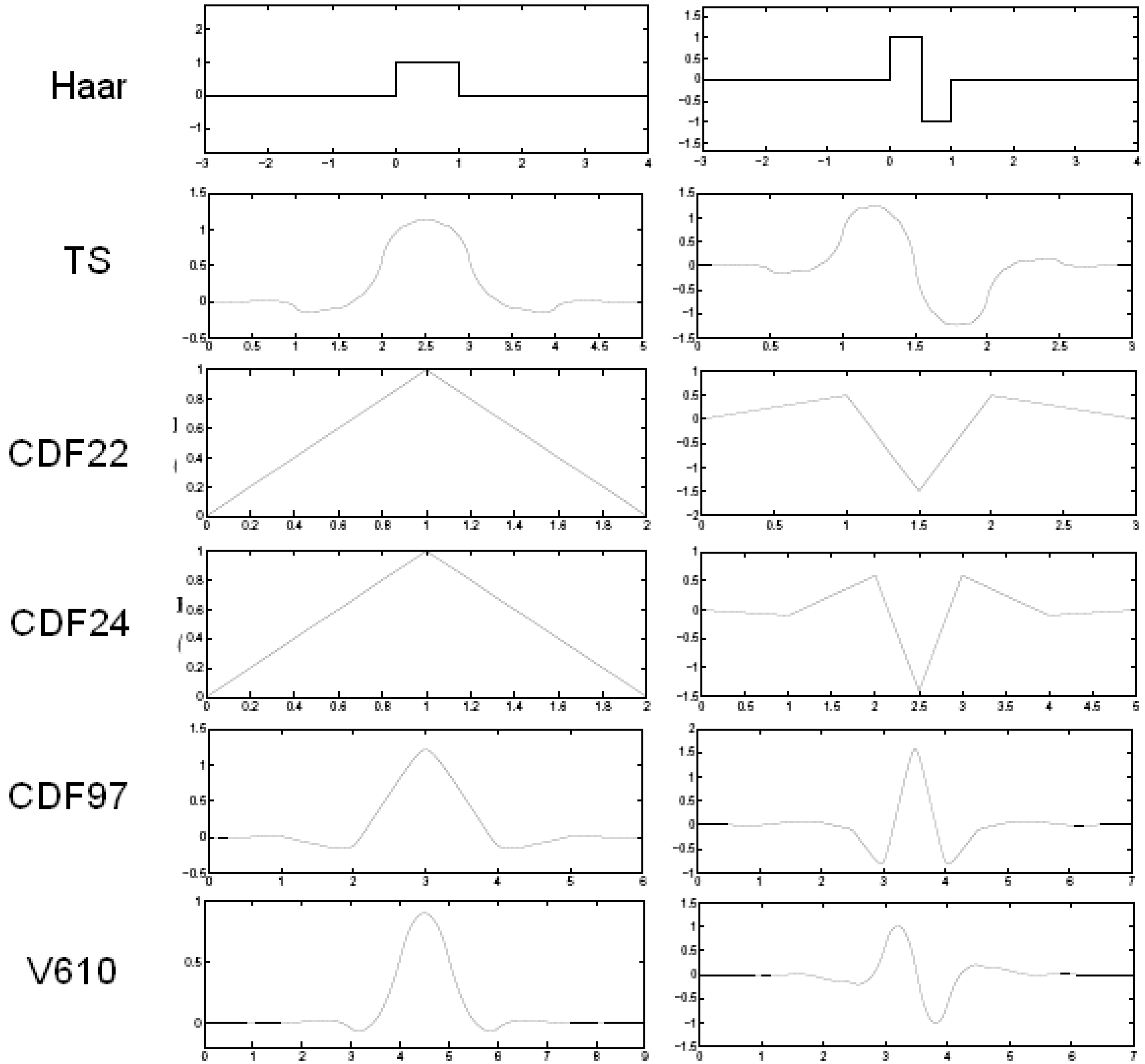
$$\text{CDF97}\quad\begin{cases} H_0(z) = 0.03783(1+z^{-8}) - 0.02385(z^{-1}+z^{-7}) - 0.1106(z^{-2}+z^{-6}) \\ \qquad + 0.3774(z^{-3}+z^{-5}) + 0.8527z^{-4} \\ H_1(z) = 0.06454(1+z^{-6}) - 0.04069(z^{-1}+z^{-5}) - 0.4181(z^{-2}+z^{-4}) \\ \qquad + 0.7885z^{-3} \end{cases}$$

$$\text{V610}\quad\begin{cases} H_0(z) = -0.1291(1+z^{-5}) + 0.04770(z^{-1}+z^{-4}) + 0.7885(z^{-2}+z^{-3}) \\ H_1(z) = -0.01891(1-z^{-9}) + 0.006989(z^{-1}-z^{-8}) + 0.06724(z^{-2}-z^{-7}) \\ \qquad + 0.1334(z^{-3}-z^{-6}) - 0.6151(z^{-4}-z^{-5}) \end{cases}$$

$$\text{MIT97}\quad\begin{cases} H_0(z) = \frac{\sqrt{2}}{64}(1+z^{-8}) - \frac{8\sqrt{2}}{64}(z^{-2}+z^{-6}) + \frac{16\sqrt{2}}{64}(z^{-3}+z^{-5}) + \frac{46\sqrt{2}}{64}z^{-4} \\ H_1(z) = -\frac{\sqrt{2}}{32}(1+z^{-6}) + \frac{9\sqrt{2}}{32}(z^{-2}+z^{-4}) - \frac{16\sqrt{2}}{32}z^{-3} \end{cases}$$

$$\text{BCW3}\quad\begin{cases} H_0(z) = -\frac{1}{256\sqrt{2}}(1+z^{-12}) + \frac{18}{256\sqrt{2}}(z^{-2}+z^{-10}) - \frac{16}{256\sqrt{2}}(z^{-3}+z^{-9}) \\ \qquad - \frac{63}{256\sqrt{2}}(z^{-4}+z^{-8}) + \frac{144}{256\sqrt{2}}(z^{-5}+z^{-7}) + \frac{348}{256\sqrt{2}}z^{-6} \\ H_1(z) = -\frac{1}{16\sqrt{2}}(1+z^{-6}) + \frac{9}{16\sqrt{2}}(z^{-2}+z^{-4}) - \frac{16}{16\sqrt{2}}z^{-3} \end{cases}$$

Scaling and wavelet functions



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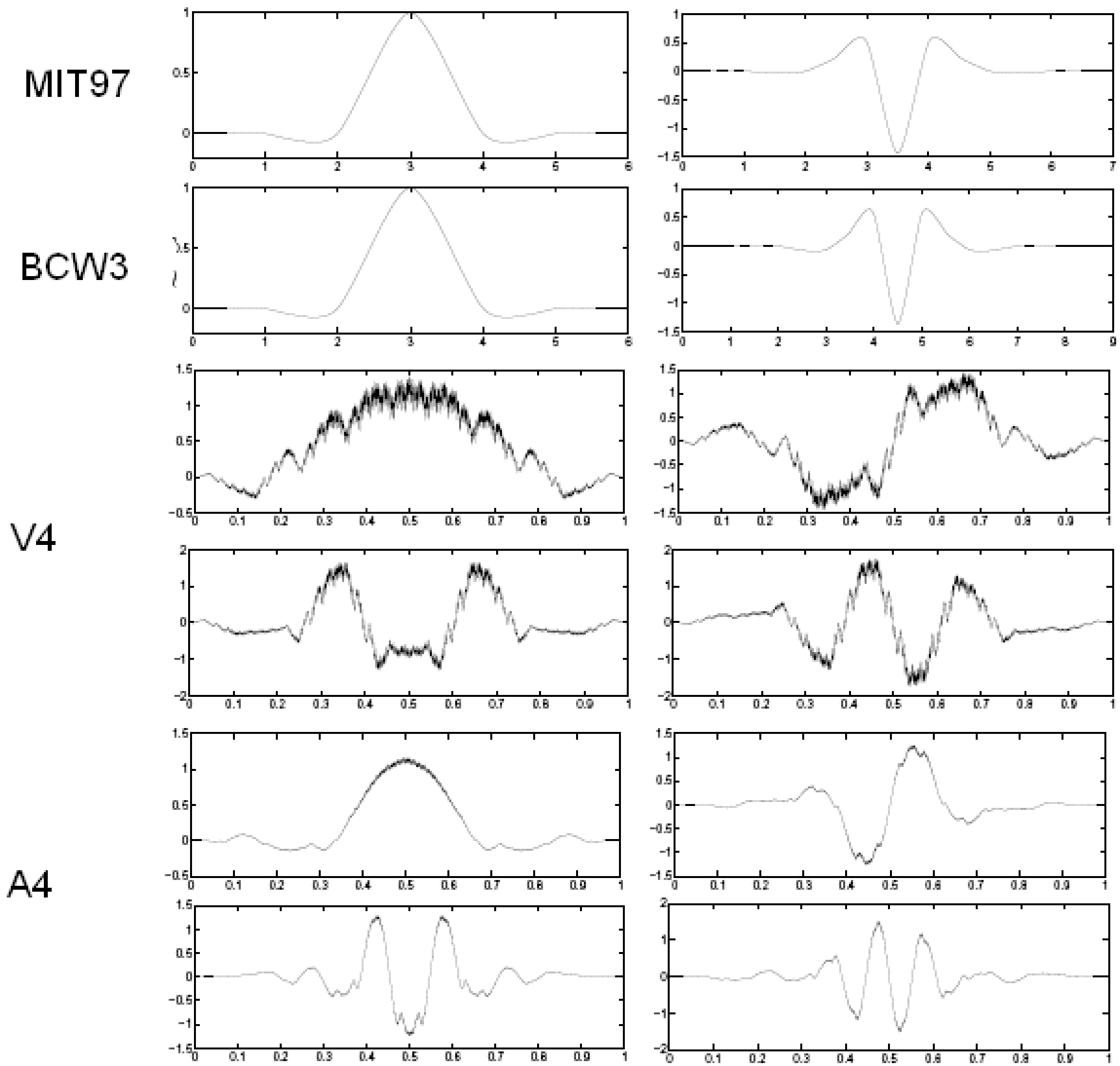


Reversible Wavelet Transforms

Reversible transforms:

- Many non-singular linear transforms that are invertible in exact arithmetic are often not invertible in finite precision arithmetic (rounding errors)
- Reversible transforms invertible in finite precision arithmetic
- Used in applications where it is undesirable to employ transforms that result in information loss
- Desirable in image compression (even in lossy!)

Reversible from nonreversible linear transforms:

- Clever use of quantization to modify the original transform so that it can be computed using finite precision arithmetic while preserving invertibility and symmetry
- Quantization makes the reversible transform nonlinear
- Approximates the original transform
- For Images, order in which row and columns are transformed is important (nonlinear!)

Approach:

- Ad hoc methods: S transform, RTS transform, S+P transform
- Systematic method: Lifting [Sweldens, 1996]

Reversible transform example

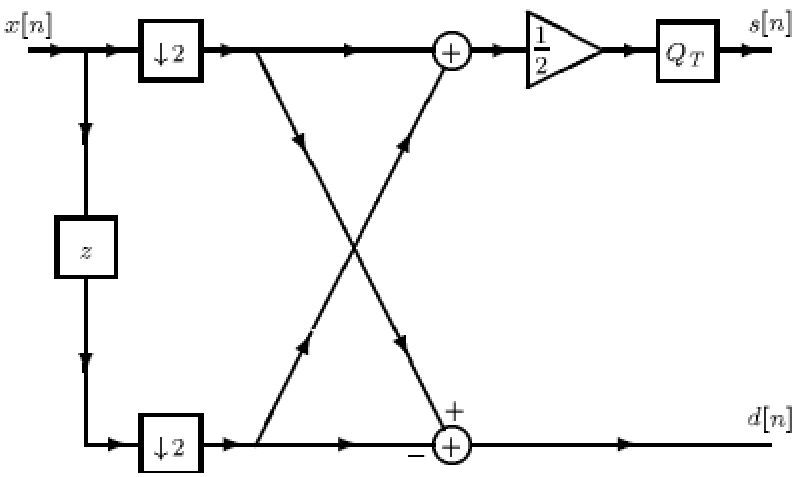
S Transform:

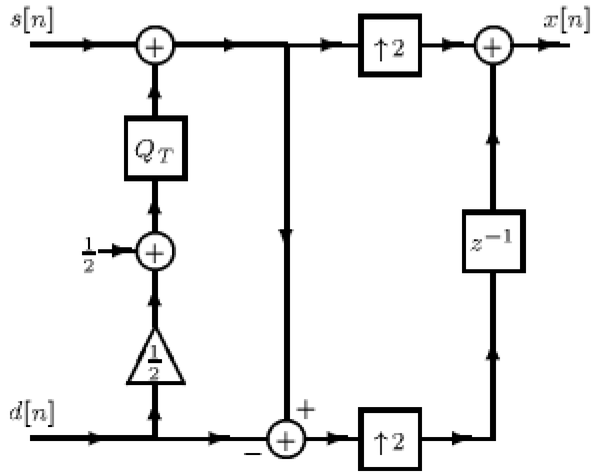
Forward:

$$s[n] = \left\lfloor \tfrac{1}{2}(x[2n] + x[2n+1]) \right\rfloor$$
$$d[n] = x[2n] - x[2n+1]$$

Inverse:

$$x[2n] = s[n] + \left\lfloor \tfrac{1}{2}(d[n] + 1) \right\rfloor$$
$$x[2n+1] = x[2n] - d[n]$$





- Maps integers to integers
- Any two numbers can be unambiguously determined from their sum and difference
- Sum and difference of any two integers have the same parity

Lifting

- Swelden (1996) proposed Lifting for converting nonreversible transforms into reversible transforms

- **Forward transform**

  Split : Decompose input sequence into multiple new sequences

  Predict : Numbers from one sequence are used to modify the values in another.

  Scale : Normalization

- **Inverse transform:** Maps the new sequences to the original sequence

  Scale

  Predict

  Join

W. Sweldens. The lifting scheme: A custom-design construction of biorthogonal wavelets. *Applied and Computational Harmonic Analysis*, 3(2):186–200, 1996. Available from `ftp://ftp.math.sc.edu/pub/imi_94/imi94_7.ps`.

Lifting Realization of a QMF bank

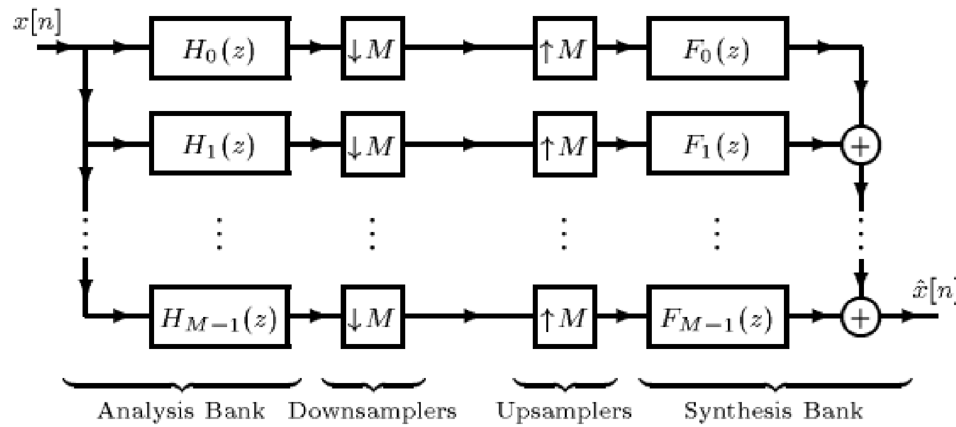
- M-Channel QMF Bank
- Polyphase representation

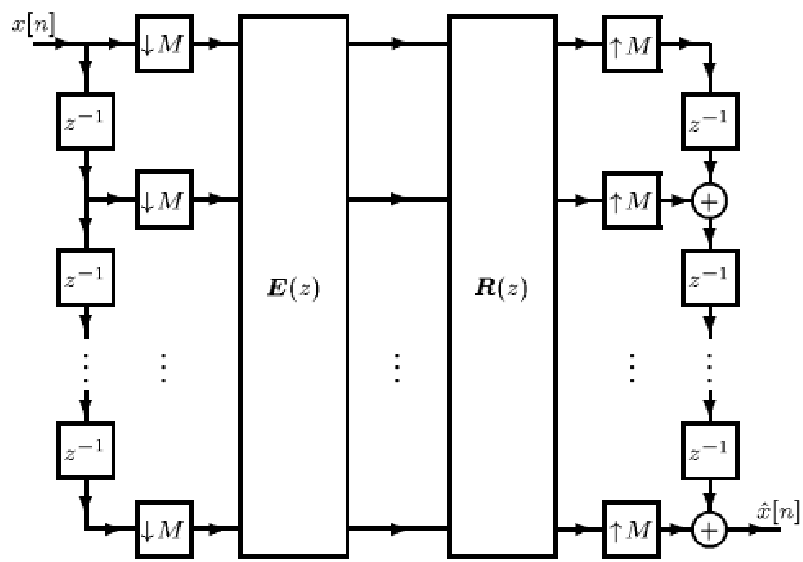
$$G(z) = \sum_{n=-\infty}^{\infty} g[n] z^{-n}$$

$$G(z) = \sum_{l=0}^{M-1} z^{a_l} B_l(z^M)$$

$$B_l(z) = \sum_{n=-\infty}^{\infty} g[Mn - a_l] z^{-n}$$

- Mathematically convenient while dealing with multi rate systems
- Computationally efficient realization





Lifting Factorization

- Factorize the analysis and synthesis polyphase matrices **E(z)** and **R(z)**

Factors: (M x **M)**

Scaling (S(i;k)) : Multiplies row/column **i** by quantity **k**

Adding (A(i,j;k)) : Adds **k** times row **j** to row **i** or **k** times column **i** to column **j**

Example: (**M** = 3)

$$S(2,k) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad A(1,3;k) = \begin{pmatrix} 1 & 0 & k \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Factorize using a matrix Euclidean algorithm:

$$\boldsymbol{E}(z) = \boldsymbol{S}_{\sigma-1} \cdots \boldsymbol{S}_1 \boldsymbol{S}_0 \boldsymbol{A}_{\lambda-1}(z) \cdots \boldsymbol{A}_1(z) \boldsymbol{A}_0(z)$$

$$\boldsymbol{R}(z) = \boldsymbol{A}_0^{-1}(z) \boldsymbol{A}_1^{-1}(z) \cdots \boldsymbol{A}_{\lambda-1}^{-1}(z) \boldsymbol{S}_0^{-1} \boldsymbol{S}_1^{-1} \cdots \boldsymbol{S}_{\sigma-1}^{-1} \;=\; \boldsymbol{E}^{-1}(z)$$

Lifted transform example

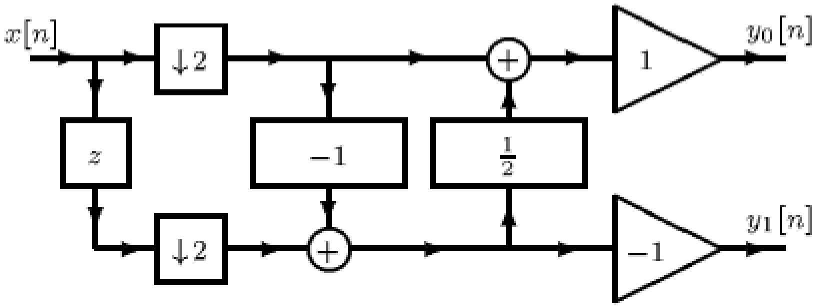
$$H_0(z) = \tfrac{1}{2}(1+z) \qquad F_0(z) = 1+z$$
$$H_1(z) = 1-z \qquad F_1(z) = \tfrac{1}{2}(1-z)$$

$$\boldsymbol{E}(z) = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & -1 \end{bmatrix}$$

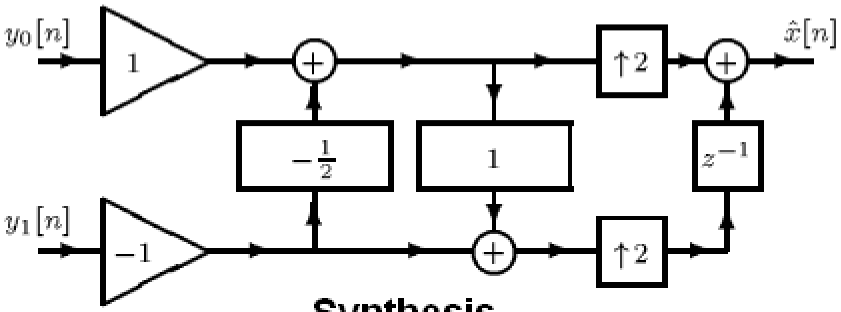
$$\det \boldsymbol{E}(z) = -1$$

$$\boldsymbol{E}(z) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$
$$= \mathcal{S}(1;-1)\mathcal{A}(0,1;\tfrac{1}{2})\mathcal{A}(1,0;-1)$$

$$\boldsymbol{R}(z) = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$
$$= \mathcal{A}(1,0;1)\mathcal{A}(0,1;-\tfrac{1}{2})\mathcal{S}(1;-1)$$



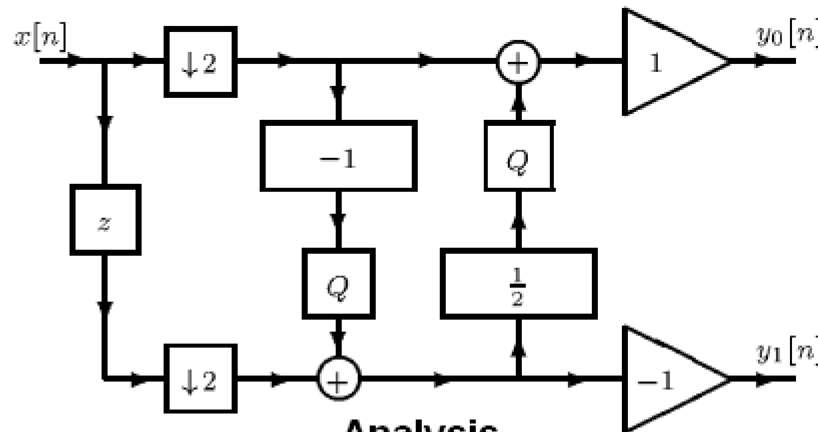
Analysis



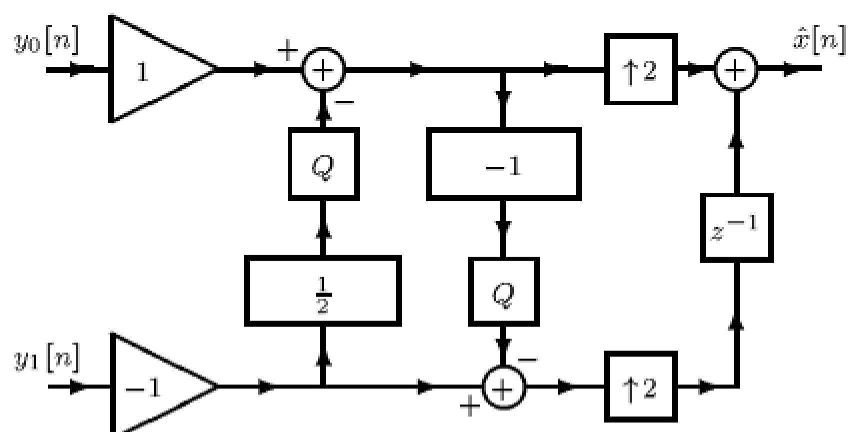
Synthesis

Reversible from Lifted Transform

- Add Quantizers
- Maps integers to integers
- Computation uses fixed point arithmetic
- Inverse transform has same computational complexity as forward transform



Analysis



Synthesis

Reversible wavelet transforms

- After converting the linear wavelet transforms to reversible transforms…

Haar†	$\boldsymbol{E}(z) = \mathcal{A}(1,0;1-\sqrt{2}) \cdot \mathcal{A}(0,1;\frac{1}{\sqrt{2}}) \cdot \mathcal{A}(1,0;1-\sqrt{2})$
TS†	$\boldsymbol{E}(z) = \mathcal{S}(0;\sqrt{2}) \cdot \mathcal{S}(1;\frac{1}{\sqrt{2}}) \cdot \mathcal{A}(1,0;-\frac{1}{4}[1-z^{-2}]) \cdot \mathcal{A}(0,1;\frac{1}{2}z) \cdot \mathcal{A}(1,0;-z^{-1})$
CDF22‡	$\boldsymbol{E}(z) = \mathcal{S}(0;\sqrt{2}) \cdot \mathcal{S}(1;\frac{1}{\sqrt{2}}) \cdot \mathcal{A}(0,1;\frac{1}{4}[1+z^{-1}]) \cdot \mathcal{A}(1,0;-\frac{1}{2}[z+1])$
CDF24‡	$\boldsymbol{E}(z) = \mathcal{S}(0;\sqrt{2}) \cdot \mathcal{S}(1;\frac{1}{\sqrt{2}}) \cdot \mathcal{A}(0,1;-\frac{3}{64}[z+z^{-2}] + \frac{19}{64}[1+z^{-1}]) \cdot \mathcal{A}(1,0;-\frac{1}{2}[1+z])$
CDF97‡	$\boldsymbol{E}(z) = \mathcal{S}(0;1.1496044) \cdot \mathcal{S}(1;0.86986445) \cdot \mathcal{A}(0,1;0.44350685[1+z^{-1}]) \cdot \mathcal{A}(1,0;0.88291108[z+1]) \cdot \mathcal{A}(0,1;-0.052980119[1+z^{-1}]) \cdot \mathcal{A}(1,0;-1.5861343[z+1])$
V610†	$\boldsymbol{E}(z) = \mathcal{S}(0;1.4142136) \cdot \mathcal{S}(1;-0.70710678) \cdot \mathcal{A}(1,0;-0.29306679[z-z^{-1}] - 0.39453543) \cdot \mathcal{A}(0,1;0.87491869) \cdot \mathcal{A}(1,0;-0.090074735z - 0.27022385) \cdot \mathcal{A}(0,1;-0.42797992 - 0.11953214z^{-1}) \cdot \mathcal{A}(1,0;-0.36953625)$
MIT97‡	$\boldsymbol{E}(z) = \mathcal{S}(0;\sqrt{2}) \cdot \mathcal{S}(1;\frac{1}{\sqrt{2}}) \cdot \mathcal{A}(0,1;\frac{1}{4}[1+z^{-1}]) \cdot \mathcal{A}(1,0;\frac{1}{16}[z^2+z^{-1}] - \frac{9}{16}[z+1])$
BCW3‡	$\boldsymbol{E}(z) = \mathcal{S}(0;\sqrt{2}) \cdot \mathcal{S}(1;\frac{1}{\sqrt{2}}) \cdot \mathcal{A}(0,1;-\frac{1}{32}[z+z^{-2}] + \frac{9}{32}[1+z^{-1}]) \cdot \mathcal{A}(1,0;\frac{1}{16}[z^2+z^{-1}] - \frac{9}{16}[z+1])$

Transform performance evaluation

Lossless Results

- Uniformity and smoothness largely determine which transform is most effective
- BCW3 and MIT97 wavelets functions are very smooth and so they are effective for very smooth images
- Due to discontinuities in the first order derivatives, CDF22 and CDF24 are effective for images with sharp transitions

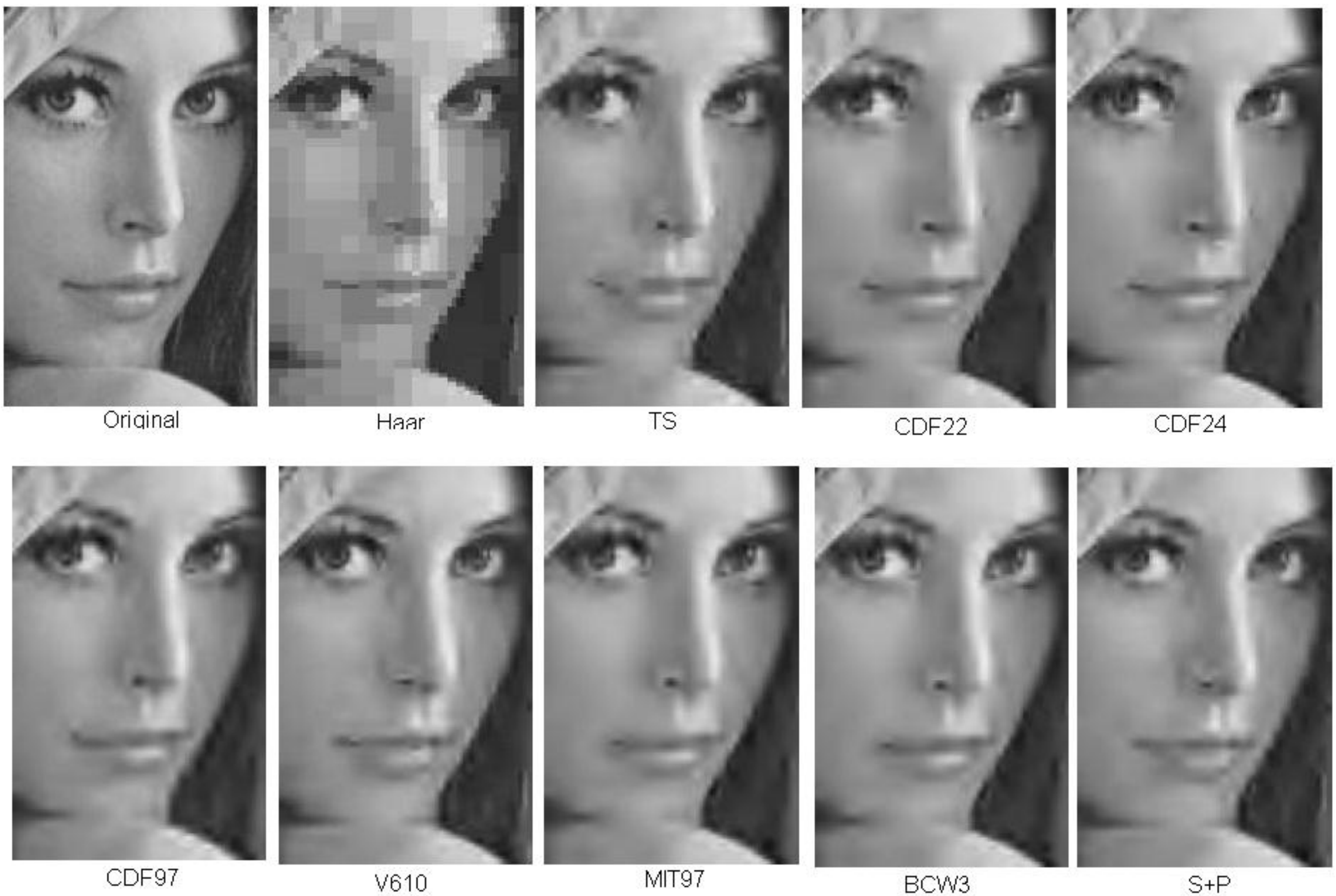
Image	CR								
	Haar	TS	CDF22	CDF24	CDF97	V610	MIT97	BCW3	S+P
air1	1.403	1.447	1.468	1.461	1.468	1.464	1.476	1.476	1.480
air2	1.726	1.810	1.869	1.860	1.819	1.796	1.891	1.889	1.881
airplane	2.317	2.326	2.409	2.401	2.264	2.318	2.385	2.389	2.389
barb	1.556	1.657	1.686	1.692	1.710	1.708	1.724	1.733	1.712
bike3	1.651	1.688	1.728	1.719	1.706	1.695	1.725	1.725	1.715
chart_s	2.320	2.329	2.433	2.412	2.272	2.283	2.388	2.385	2.402
cmpnd1	4.490	3.559	3.714	3.407	2.946	2.988	3.103	3.041	3.504
cmpnd2	4.272	3.457	3.713	3.395	2.949	2.853	3.117	3.057	3.431
cr	1.864	1.909	1.930	1.928	1.919	1.924	1.924	1.928	1.911
ct	2.495	2.906	2.975	2.941	3.037	2.991	3.191	3.179	3.120
eafbcmpnd	2.445	1.701	1.851	1.706	1.475	1.480	1.524	1.501	1.697
express1	3.133	2.239	2.515	2.388	2.003	1.953	2.187	2.161	2.263
finger	1.272	1.431	1.439	1.432	1.451	1.432	1.477	1.475	1.445
france	5.328	3.245	3.759	3.452	2.527	2.561	2.945	2.900	3.228
gold	1.588	1.633	1.675	1.670	1.655	1.658	1.675	1.676	1.662
hotel	1.623	1.639	1.687	1.684	1.667	1.665	1.686	1.687	1.672
lax	1.358	1.358	1.371	1.368	1.366	1.368	1.366	1.368	1.360
lena	1.740	1.835	1.875	1.870	1.875	1.859	1.891	1.893	1.881
library	1.460	1.371	1.401	1.382	1.337	1.337	1.352	1.347	1.359
mandrill	1.283	1.315	1.329	1.327	1.330	1.331	1.333	1.335	1.332
medical_93	2.588	2.768	2.941	2.920	2.706	2.775	3.004	2.998	2.953
molecule	3.903	3.808	4.030	4.004	3.610	3.445	3.927	3.930	3.960
mri	1.722	1.809	1.845	1.838	1.850	1.860	1.879	1.879	1.885
us	2.567	2.424	2.539	2.484	2.202	2.253	2.429	2.415	2.507
xray1	2.696	2.857	3.103	3.088	2.812	2.751	3.156	3.155	3.092

Lossy Results

- Earlier observations on transform effectiveness is true for most cases
- At very high compression ratios, the best transform is hard to predict interms of PSNR.
- Subjective tests are useful

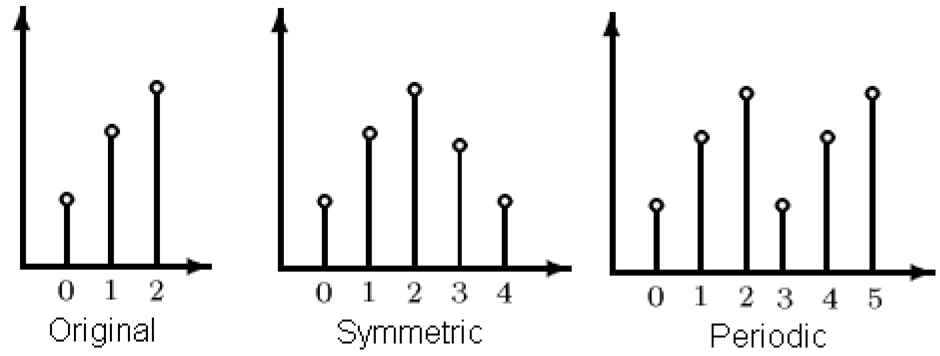
Image	CR	PSNR (dB)								
		Haar	TS	CDF22	CDF24	CDF97	V610	MIT97	BCW3	S+P
barb	8	32.78	34.63	34.40	34.61	35.28	35.13	35.15	35.34	34.89
	16	28.69	29.98	30.21	30.47	30.62	30.81	30.62	30.91	30.13
	32	25.43	26.36	26.46	26.76	26.93	26.99	26.59	26.83	26.36
	64	23.56	24.13	24.04	24.17	24.77	24.22	24.08	24.23	24.38
	128	22.60	23.19	23.40	23.51	23.39	23.49	23.27	23.39	23.27
chart_s	8	40.22	40.69	41.30	41.27	40.07	40.29	41.04	41.11	40.95
	16	33.21	34.17	34.02	33.81	34.98	33.88	34.30	34.39	35.09
	32	28.52	29.80	29.41	29.46	30.43	29.73	29.81	29.90	30.40
	64	25.05	26.98	26.62	26.65	27.53	26.72	26.97	27.07	27.42
	128	23.13	24.45	23.92	24.03	24.35	24.08	24.40	24.51	24.32
cmpnd1	8	45.98	41.01	41.31	40.74	39.40	39.46	39.88	39.73	40.97
	16	36.61	32.80	30.75	30.36	31.96	31.40	30.67	30.61	32.16
	32	25.42	25.06	24.43	24.43	24.98	24.50	24.29	24.41	24.75
	64	21.19	20.91	20.45	20.58	21.15	20.40	20.54	20.64	20.78
	128	18.32	18.60	19.42	19.66	19.23	19.34	19.43	19.45	18.90
finger	8	26.56	30.70	29.63	29.56	31.01	29.97	30.94	31.00	30.33
	16	23.74	26.93	26.08	26.17	27.06	26.55	26.80	26.94	26.81
	32	22.08	23.74	22.92	23.02	23.44	23.43	23.41	23.55	23.40
	64	20.28	21.02	21.21	21.20	21.16	21.74	21.36	21.53	21.04
	128	19.51	20.03	19.49	19.67	19.83	19.83	19.40	19.54	20.05
lena	8	37.07	38.76	39.03	39.02	38.51	38.63	39.15	39.22	39.02
	16	33.34	35.49	36.00	36.04	35.66	35.78	36.15	36.23	35.79
	32	30.50	32.31	33.03	33.14	32.79	32.93	33.17	33.27	32.84
	64	27.98	29.36	30.21	30.41	29.91	30.19	30.24	30.35	30.02
	128	25.59	26.95	27.52	27.72	27.54	27.49	27.45	27.57	27.60
molecule	8	43.49	48.43	47.86	47.85	44.89	45.31	47.94	47.97	48.24
	16	39.60	43.66	44.25	44.24	42.24	42.24	44.08	44.09	43.69
	32	35.12	40.14	41.00	41.00	39.94	38.94	41.05	41.09	40.66
	64	31.42	36.25	37.64	37.30	36.54	35.86	37.59	37.70	36.81
	128	28.23	32.55	33.04	33.01	33.00	32.32	34.00	33.99	32.86

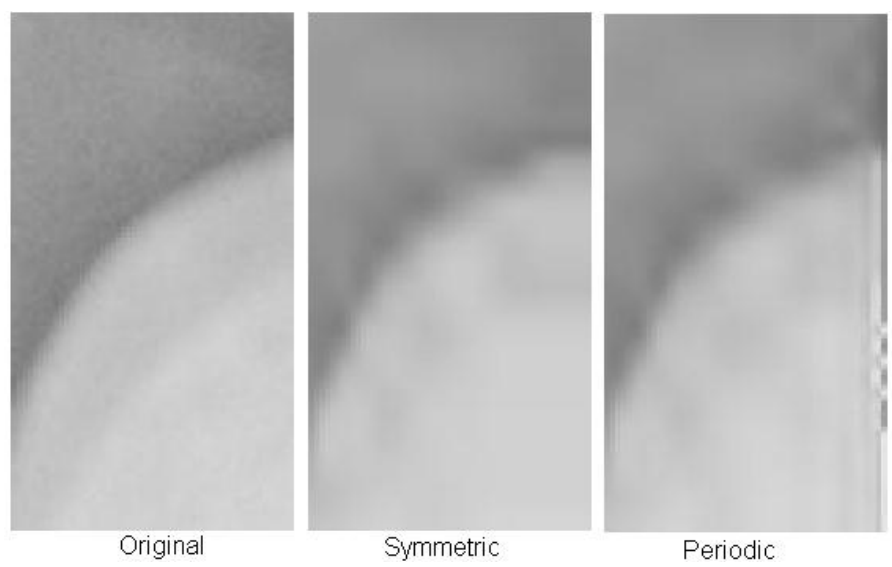
Lossy reconstruction: CR = 64



Periodic Vs Symmetric Extension

- Periodic extension produces jump discontinuities while symmetric extension does not
- Periodic extension can result in expansive transforms while symmetric expansion allows for nonexpansive transforms
- Hence, symmetric expansion is superior to periodic expansion





Multi-Transform approach

- No single transform yields best results for all classes of images
- Include multiple transforms in the compression system, and dynamically pickup the most optimal transform based on the image.

$$s = 100N_s/N$$

$$u = 100N_u/N$$

Classify images based on

- Smoothness
- Uniform intensity

N_s is the number of d_i satisfying $|d_i| \geq R/2$

N_u is the number of d_i satisfying $d_i = 0$.

Transform Selection (Example):

Image	Wavelet
Very smooth	BCW3
Moderate smoothness Large uniform intensity regions	CDF22
Non smooth variation	Haar

Multi-Transform Results

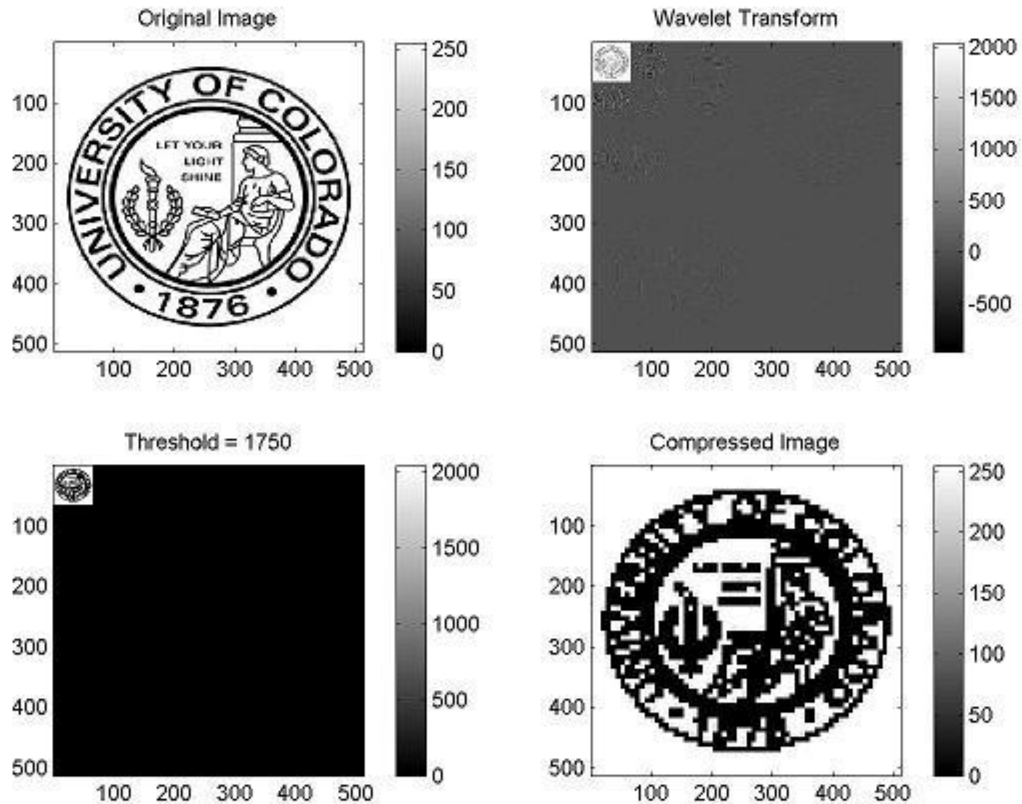
Pick the best of all worlds!

Image	Compression Ratio							
	Haar	CDF22	CDF24	CDF97	MIT97	BCW3	S+P	Proposed Method with/without Haar
air1	1.403	1.468	1.461	1.468	1.476	1.476	1.480	1.476
air2	1.726	1.869	1.860	1.819	1.891	1.889	1.881	1.889
airplane	2.317	2.409	2.401	2.264	2.385	2.389	2.389	2.409
barb	1.556	1.686	1.692	1.710	1.724	1.733	1.712	1.733
bike3	1.651	1.728	1.719	1.706	1.725	1.725	1.715	1.728
chart_s	2.320	2.433	2.412	2.272	2.388	2.385	2.402	2.433
cmpnd1	4.490	3.714	3.407	2.946	3.103	3.041	3.504	4.490/3.714
cmpnd2	4.272	3.713	3.395	2.949	3.117	3.057	3.431	4.272/3.713
cr	1.864	1.930	1.928	1.919	1.924	1.928	1.911	1.928
ct	2.495	2.975	2.941	3.037	3.191	3.179	3.120	3.179
eafbcmpnd	2.445	1.851	1.706	1.475	1.524	1.501	1.697	2.445/1.851
express1	3.133	2.515	2.388	2.003	2.187	2.161	2.263	3.133/2.515
finger	1.272	1.439	1.432	1.451	1.477	1.475	1.445	1.475
france	5.328	3.759	3.452	2.527	2.945	2.900	3.228	5.328/3.759
gold	1.588	1.675	1.670	1.655	1.675	1.676	1.662	1.676
hotel	1.623	1.687	1.684	1.667	1.686	1.687	1.672	1.687
lax	1.358	1.371	1.368	1.366	1.366	1.368	1.360	1.368
lena	1.740	1.875	1.870	1.875	1.891	1.893	1.881	1.893
library	1.460	1.401	1.382	1.337	1.352	1.347	1.359	1.460/1.401
mandrill	1.283	1.329	1.327	1.330	1.333	1.335	1.332	1.335
medical_93	2.588	2.941	2.920	2.706	3.004	2.998	2.953	2.998
molecule	3.903	4.030	4.004	3.610	3.927	3.930	3.960	4.030
mri	1.722	1.845	1.838	1.850	1.879	1.879	1.885	1.879
us	2.567	2.539	2.484	2.202	2.429	2.415	2.507	2.567/2.539
xray1	2.696	3.103	3.088	2.812	3.156	3.155	3.092	3.103
Mean	2.352	2.291	2.233	2.078	2.190	2.180	2.234	2.477/2.308

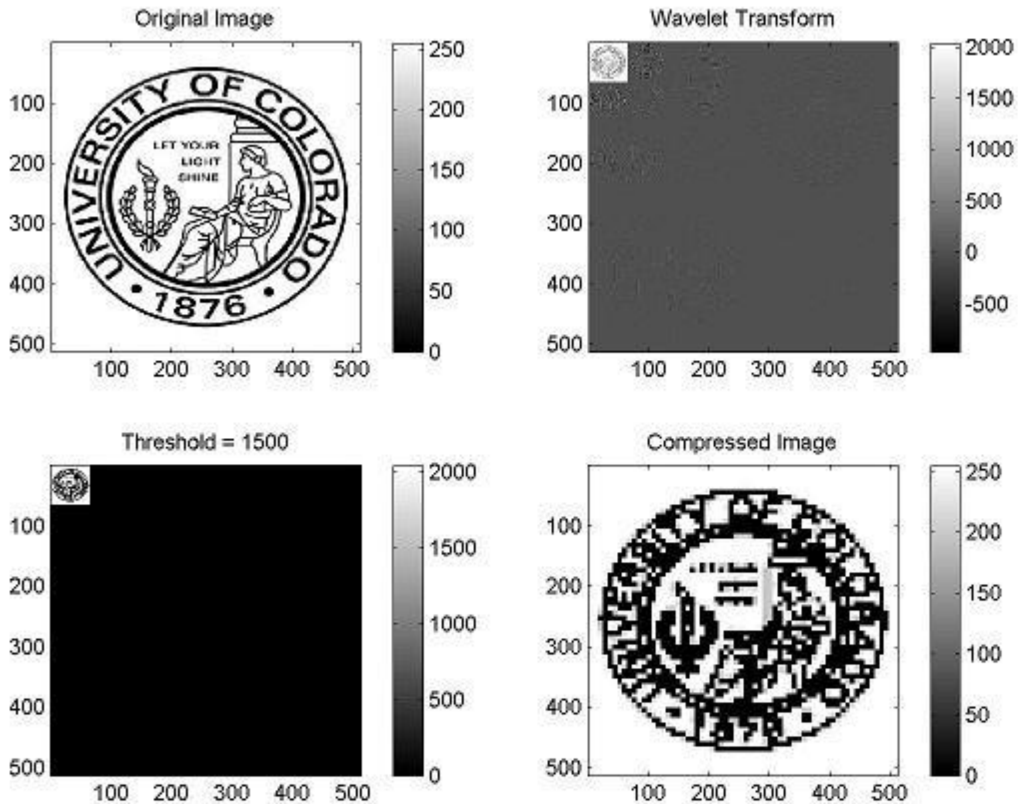
Conclusion

- Symmetric wavelets are efficient for embedded image compression
- Lifting can produce reversible transforms that well approximate their parent linear transforms
- Effectiveness of a transform in image dependant, and hence, multi transform approach is useful in a generalized compression system that has no apriori information about the images to be compressed

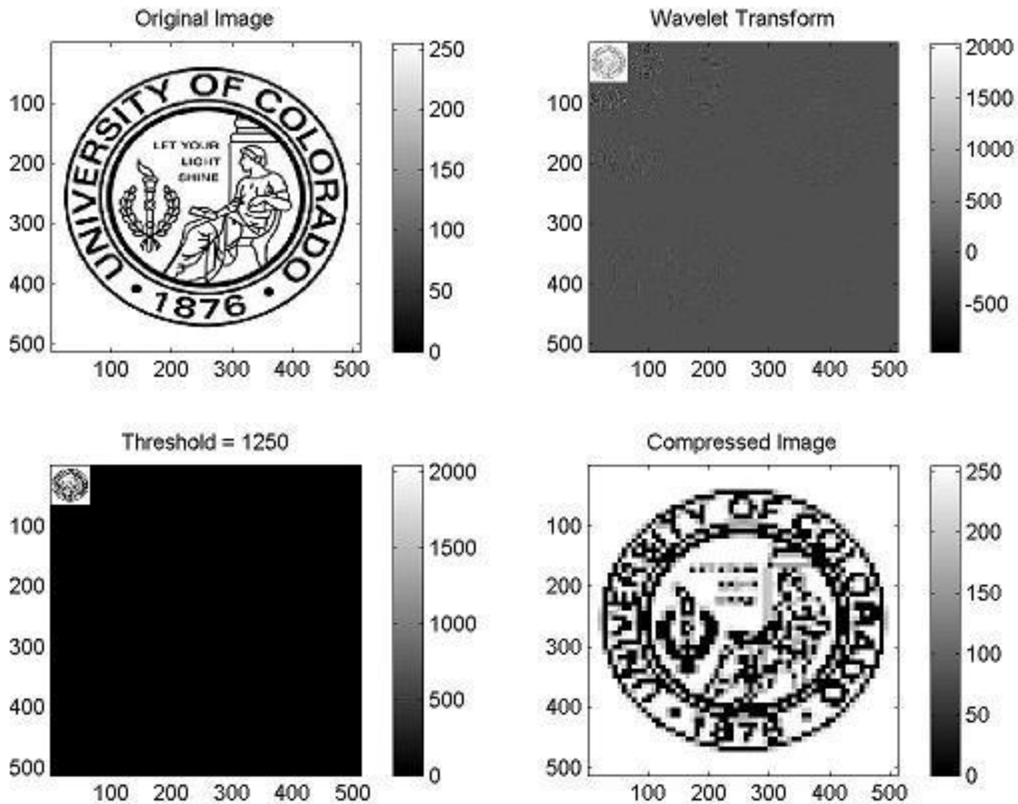
Demo


Original Image
UNIVERSITY OF COLORADO
LET YOUR LIGHT SHINE
1876
Wavelet Transform
Threshold = 1750
Compressed Image

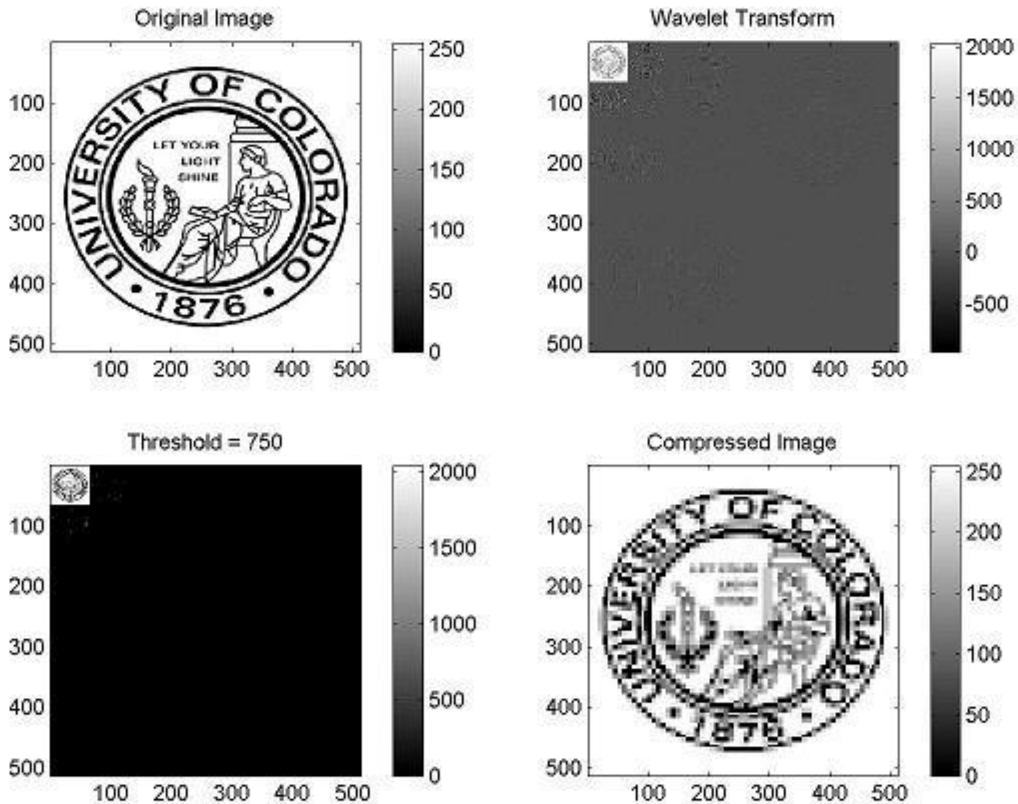
Threshold = 1500



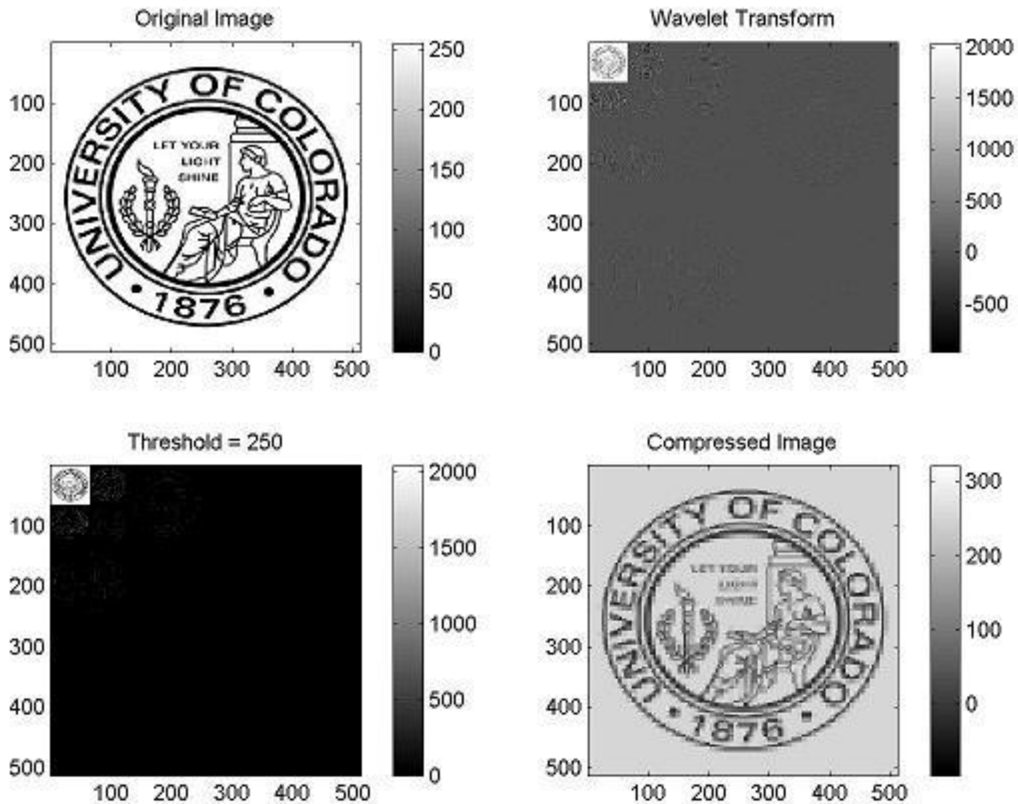
Threshold = 1250



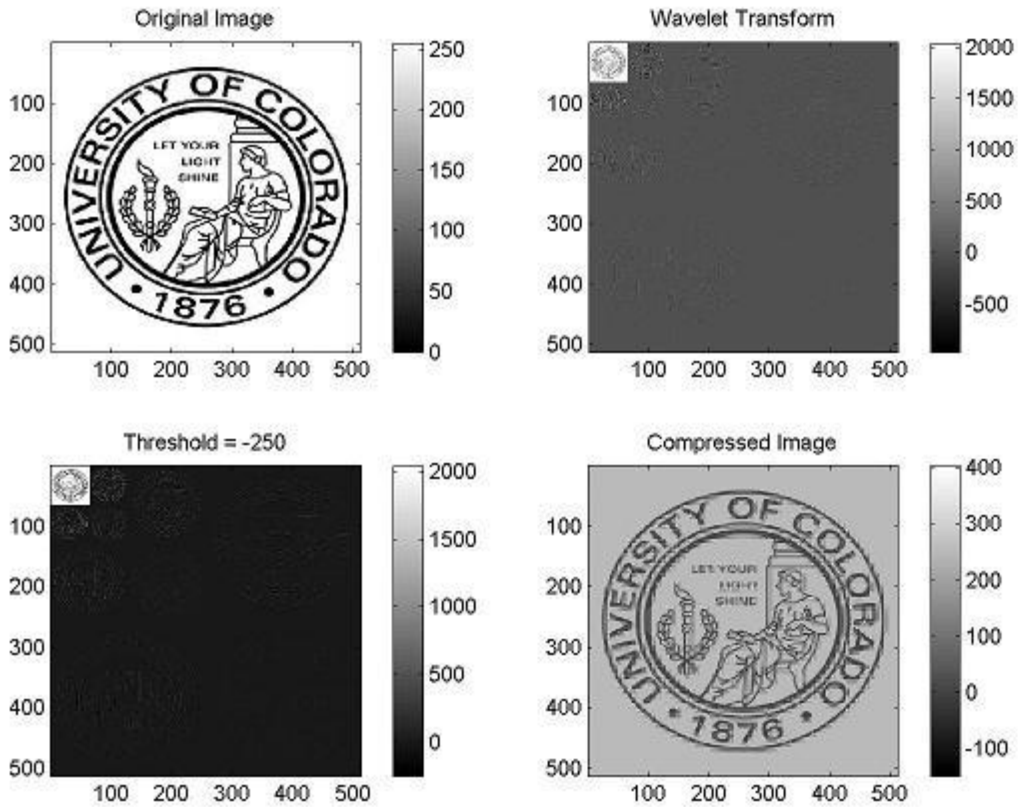
Threshold = 750



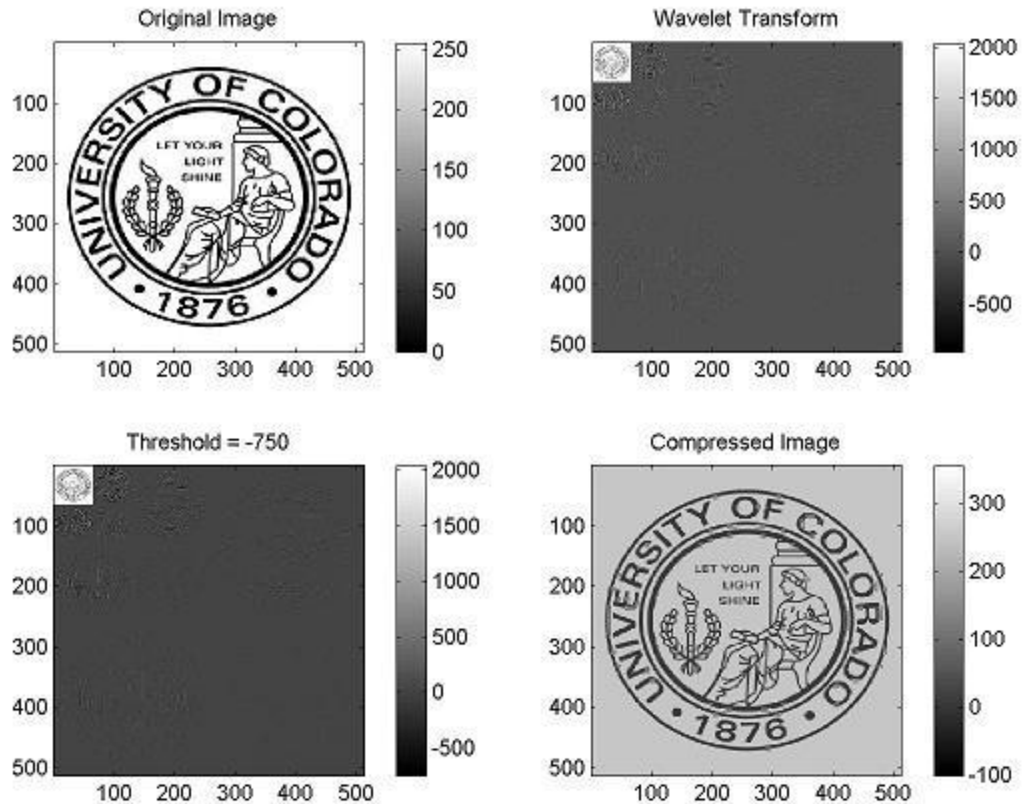
Threshold = 250



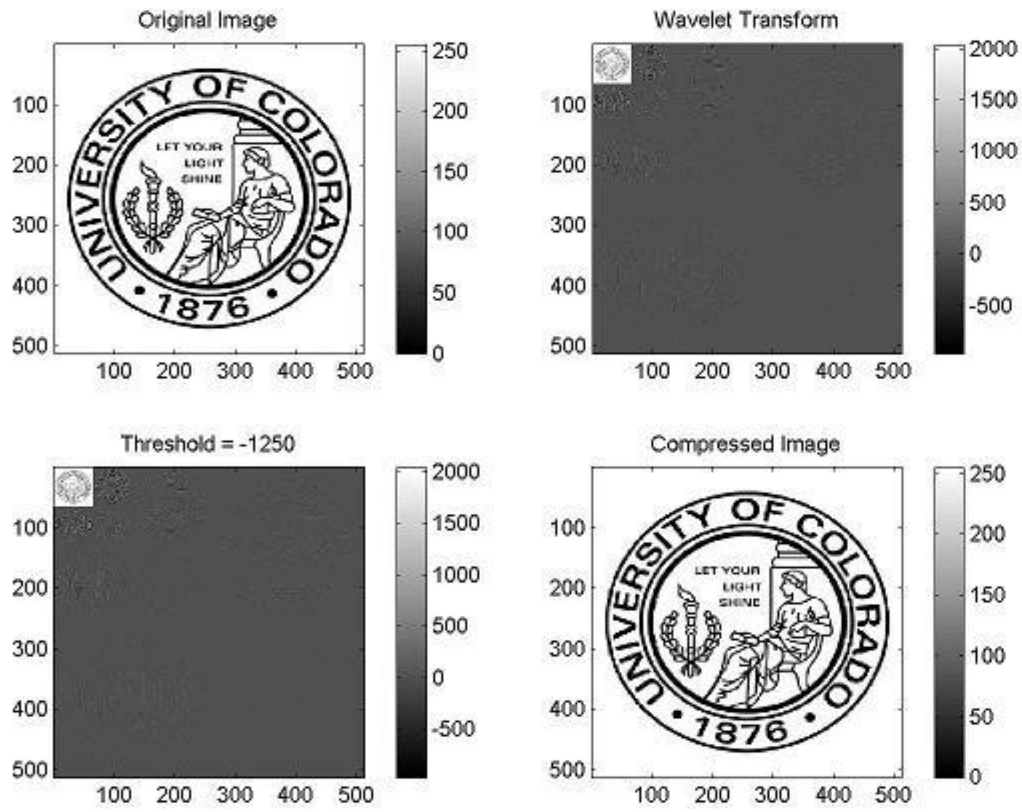
Threshold = -250



Threshold = -750



Threshold = -1250



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Acknowledgements

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- Thank You!